PART A: QUESTIONS 1-8.

(1) (a) Law of Large numbers.

⇒ The law of large numbers states that a sample size increases, its mean gets closer to the average of the whole population.

Example: Rolling a dice.

Expected value of a dice is: $\frac{1+2+3+4+5+6}{6} = 3.5$.

If we roll a dice only three times, the average of the obtained result may be far from the expected value.

eg, $4, 4, 6 = \frac{14}{3} = 4.67$

According to the law of large numbers, if we roll a dice a larger number of times, the average result will be closer to ~~£~~ 3.5.

(b) The non-existent law of averages.

It states that frequencies of events which have the same likelihood of occurrence even out, give enough trials or instances.

Example: Batting averages in baseball.

If a player has a batting average of 0.25, then he can be expected to get a hit on one out of 4 at-bats in the long term. During a 'streak', he might get a hit in 4 out of ten at bats and during 'slumps' he can get a hit on only One out of 10 at-bats. If the law of average is used, when the hitter is 'slumpling' it is said that 'he is due for a hit' so suggesting that on each at-bat his chances are better than 1 of 4.

(c) Fundamental counting principal ('for 'and' & 'or')

It is a way to figure the number of outcomes in a probability problem. ie, You ~~can~~ multiply events together to get the total number of outcomes.

Example.

If you take a survey with five 'yes' or 'no' answers. How many different ways could you complete the survey?

soln.

There are two choices for each question (Yes, No). So the total number of possible ways to answer is:

$2 \times 2 \times 2 \times 2 \times 2 = 32$.

(d) Permutation:

It is a mathematical technique that determines the ~~total~~ number of possible arrangements in a set when order matter.

ie, $P(n, r) = \frac{n!}{(n-r)!}$

Example.

how many times does abbccc be arranged?

$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 (3 \cdot 2)} = 60.$

(e) Combination.

It is a way of selecting items from a collection, such that the order of selection does not matter.

~~ie~~ formula: ${}_nC_r = \binom{n}{r} = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n-r)!}$.

Example.

How many combination we can write the vowels of word GREAT

$= {}_5C_2 = \frac{5!}{2!(5-2)!} = \frac{5 \times 4 \times \cancel{3 \times 2 \times 1}}{(2 \times 1)\cancel{(3 \times 2 \times 1)}} = 10.$

2. Chance of winning a prize:

$= E.V = \frac{1+2+3+4}{4} = \frac{10}{4} = 2.5 \neq 0.25$

$\frac{1}{4}$ is the probability of winning only one price. According to the law of large numbers, if you win more prices, the average result will be closer to 2.5.

3. (a) How many ways can one boy and one girl be chosen?

~~= 10 × 18 = 180 ways to choose one boy and one girl.~~

$= P(28, 2) = \frac{28!}{26!} = \frac{28 \times 27 \times \cancel{26!}}{\cancel{26!}}$

$= 756$

(b) One boy or one girl

$= C(28, 2) = \frac{28!}{2!(26)!} = \frac{28 \times 27 \times \cancel{26!}}{2 \times \cancel{26!}}$

$= 378$

4. 12 suspects. How many times can 4 of them be arranged?

$= {}^{12}C_4 = \frac{12!}{4!(12-4)!} = \frac{12!}{(4 \times 3 \times 2 \times 1)\,\frac{\cancel{12 \times 11 \times 10 \times 9 \times 8}}{8!}}$

$= \frac{\cancel{12}^{1} \times 11 \times \cancel{10}^{5} \times 9 \times \cancel{8!}}{\cancel{24}_{2\,1} \times \cancel{8!}} = 495$ ways.

ok

5. A commitee of 5 members is to be randomly selected from 8 freshmen & 6 somophores

(a) N.o of ways selecting

3 freshmen out of 8 freshmen: $8C_3$

&

2 somophores out of 6 somophores: $6C_2$

$= 8C_3 \times 6C_2$

$= \frac{8!}{3!\,5!} \times \frac{6!}{2!\,4!} = \frac{8\times7\times6\times\cancel{5!}}{3\times2\times1\times\cancel{5!}} \times \frac{6\times5\times4!}{2\times4!}$

$= 56 \times 15$

$= 840.$

(b) Jake, is randomly ~~selected~~ chosen from the committee.

N.o of ways of selecting 5 from 14

$= 14C_5$ ways

$= 2{,}002$

N.o of ways of selecting 1 freshman from 8 freshmen.

$= 8C_1$ ways

$= 8$

$= \frac{8}{2002} = 0.00396$

$= \frac{1C_1 \times 7C_2 \times 6C_2}{840} = \frac{1\times21\times15}{840} = \frac{315}{840}$

$= 0.375$

6. ~~Think~~ Thinking about 5-card poker hands...

(a) How many hands are possible?

$= {}_{52}C_5 = 2,598,960$ possible 5-card hands.

b. How many hands with exactly 3 aces are possible?

$= {}_4C_3 \times {}_{48}C_2$.

$= 4 \times 1128 = 4,512$.

(c) How many hands with exactly 1 or 2 aces are possible?

= Hands with exactly 1 $= 4C_1 \cdot {}_{48}C_4$

$= 4 \times 194,580 = 778,320$.

Hands with exactly 2..

$= 4C_2 \cdot {}_{48}C_3$.

$= 6 \times 17296$

$= 103,776$.

$= 778,320 + 103,776$

$= 882,096$.

7. (#, \$, %, or &) as in AW52\$

(a) Number of passwords possible:

$= 26 \times 26 \times 10 \times 10 \times 4$

$= 270,400$ different passwords.

(b) The probability that a password generated randomly in the above format will contain one's first and last name initials in order:

$$= \frac{1 \times 1 \times 10 \times 10 \times 4}{270400} = \frac{400}{270400}$$

$= 0.001479.$

8. 7 - African American.

5 - Asian

6 - Hispanic.

4 - white.

$$P(\text{No Hispanics}) = \frac{\text{n.o of combinations with no hispanics.}}{\text{n.o of combination of winnow.}}$$

$7+5+6+4 = 22.$

$$= \frac{{}_{16}C_6}{{}_{22}C_6}$$

$$= \frac{8008}{74,613}$$

$\approx 0.10733.$

No. ~~White~~ But unlikely, it may still occur du to random chance.

Part B: Questions 9 - 12.

9. $\frac{36}{100} \Rightarrow$ male.

$\frac{85}{100} \Rightarrow$ satisfied.

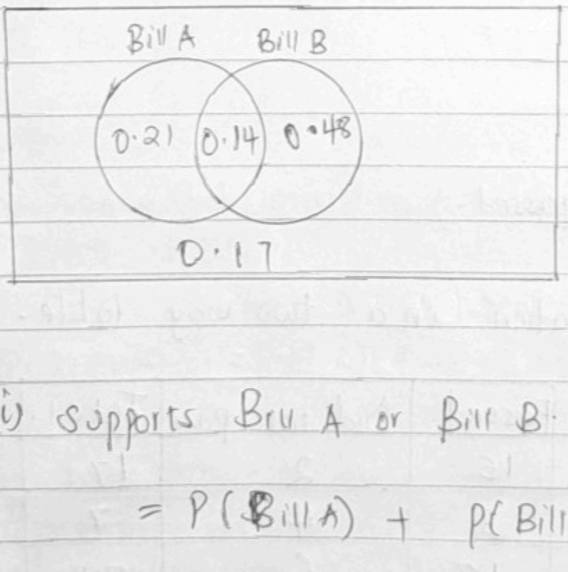
$= P(\text{male} \cap \text{unsatisfied}) = P(\text{male}) \times P(\text{satisfied} | \text{male})$

$= \frac{36}{100} \times \frac{85}{100} = \frac{3060}{10000} =$

$= 0.306.$

10. $35\% \Rightarrow$ Bill A.
$62\% \Rightarrow$ Bill B.
$14\% \Rightarrow$ Both.

(a) Draw a venn diagram.



$$\begin{array}{r} 48 \\ +14 \\ \hline 62 \\ 21 \\ \hline 83 \end{array}$$

$100 - 83 = 17.$

(b) (i) supports Bill A or Bill B.

$= P(\text{Bill A}) + P(\text{Bill B}) + P(\text{Bill A} \cap \text{Bill B})$

$= 0.21 + 0.14 + 0.48$

$= 0.83.$

ii) Supports Bill A but not Bill B.

$= P(\text{Bill A only})$
$= P(\text{Bill A}) - P(\text{Bill A} \cap \text{Bill B})$
$= 0.35 - 0.14$
$= 0.21$

iii) Supports neither Bill A nor Bill B.

$= 1 - \{P(\text{Bill A only}) + P(\text{Bill B only}) + P(\text{Bill A} \cap \text{Bill B})\}$

$= 1 - (0.21 + 0.48 + 0.14)$

$= 1 - 0.83$
$= 0.17$

11) Grand total = 24
17 - completed all
18 - passed.
15 - completed and passed.

(a) Organize this information in a two way table.

		Passed	Did not pass	Total.
Homework	Completed all	15	2	17
	Did not complete all	3	4	7
	Total.	18	6	24

(b) Probability that:
(i) The student passed the test.
$= \frac{18}{24} = 0.75$.

(ii) The student did not complete the homework.

$$= \frac{7}{24} = 0.2917$$

~~(b)~~ iii) The student completed the homework and passed the test

$$= \cancel{\frac{15}{24}} = \frac{15}{24} = 0.625.$$

iv) The student passed the test given s/he completed the homework.

$$= \cancel{\frac{15}{18}} = \cancel{0.833\dot{3}} = \frac{15}{17} = 0.88235$$

v) The student ~~passed~~ completed the homework if s/he passed the test.

$$= \frac{15}{18} = 0.833\dot{3}$$

(c) ~~vi)~~ ~~The~~ In this example, is test success independent of the homework completion? Explain.

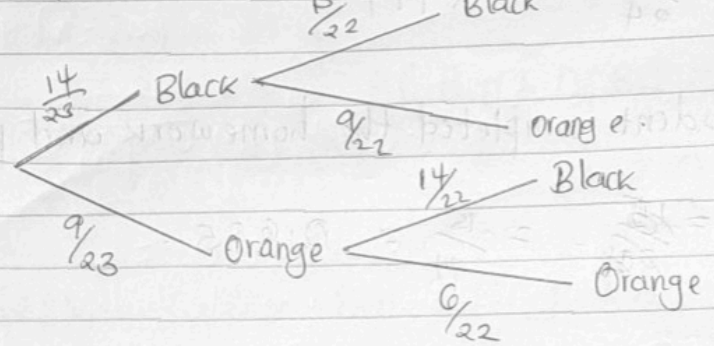
~~Test~~ Ans

Test success is not independent of homework completion.
p(success|completed) = 0.88235 and P(success) = 0.75
If the two were independent, these probabilities would be the same.
Students who completed the homework had a higher chance of success.

12. 23 Halloween candies, 14 black, 9 orange.

picks 2.

(a)



(b) Ramon has picked black. P(2nd pick is an orange)

~~P(Black)~~ = P(1st black) $= \frac{14}{23}$ P(2nd black) $= \frac{13}{22}$.

$= 1 - \frac{13}{22} = \frac{9}{22}$.

(c) i) picks an orange candy then a black candy.

$$= \frac{9}{23} \times \frac{14}{22} = \frac{126}{506}$$

$$= 0.24901$$

ii) Picks a black candy given he has already picked an orange one.

$$= \frac{14}{22}$$

iii) Picks two different coloured candies.

= P(Black and orange) or P(orange and black)

$$= \left(\frac{14}{23} \times \frac{9}{22}\right) + \left(\frac{9}{23} \times \frac{14}{22}\right)$$

$$= \frac{126}{506} + \frac{126}{506} = \frac{126}{253} = 0.49802$$

(iv) Picks no orange candies

$= \frac{14}{27}$ ~~P(Black~~

$= P(1^{st} \text{ black}) \times P(2^{nd} \text{ black})$

$= \frac{14}{23} \times \frac{13}{22} = \frac{182}{506} = 0.3597$

Part C : Questions 13 - 15.

13. 19.1% ⇒ ~~2003~~ silver.
45 luxury a month.

(a) Explain why ~~car~~ silver cars sold can ~~are~~ be considered as Bernouli?
It should be considered bernoulli trials because:
- It has only two outcomes, either will be silver or not.
- The sample is large.
- The probability of each independent will not change.

(b) Let $X \Rightarrow$ n.o of cars manufactured in 2003 were silver.

$X \sim Bin(n, p)$ $X \sim B(45, 19.1)$ $p = 0.191$

$P(X=5) = \binom{45}{5} (0.191)^5 (1-0.191)^{45-5}$ $n = 45$.

$= 0.06458$

(c) $P(X=5) = (1-0.191)^4 (0.191)$

$= 0.08181$

(c) Exactly ten of 45 luxury cars are silver?

$X \sim Bin(45, 0.191)$

$= P(X = 10)$

$= \binom{45}{10} (0.191)^{10} (1 - 0.191)^{45-10}$

$= 0.123693$

(d) At least ten of 45

$= P(X \geq 10)$

$= \sum_{x=10}^{45} \binom{45}{a} (0.191)^{x} (1 - 0.191)^{45-x}$

From binomial calculator

$= 0.35297$

(e) Mean and standard deviation.

Mean $= np$ std $= \sqrt{npq}$

mean $= 45 \times 0.191 = 8.595$

std $= \sqrt{45 \times 0.191 \times 0.809} = \sqrt{6.953355} = 2.6369$

Mean $= \underline{8.595}$ Standard deviation $= \underline{2.6369}$

14. $\cancel{n=}$ $p = 0.7$

$n = 600$

$x = 440$

let x = number of drivers that report that they wear a seat belt.

(a) Verify that a Normal model is a good approximation for the binomial model in this situation.

Given $p = 0.7$, $q = 1 - 0.7 = 0.3$. $np = 420$, $nq = 180$

Since $np \geq 10$ and $nq \geq 10$, we can use normal approximation where

$$\text{mean} = np = 420.$$

$$\cancel{\text{std}}\ \text{var} = npq = 126, \quad \text{std} = 11.225.$$

$$\therefore X \sim N(420, 126)$$

(b) Mean & std of the normal model.

$$\text{mean} = np = 420.$$

$$\text{std} = \sqrt{npq} = \sqrt{0.7 \times 0.3 \times 600} = \sqrt{126} = 11.225.$$

Mean = 420 and standard deviation = 11.225.

$$\therefore X \sim N(420, 126)$$

(c) Does the survey result convince that the survey was effective?

e need
test
he

H_0: The advertisement did not ~~advert~~ increase saftey belt use

vs

H_1: The advertisement increased the saftey belt use

is the hypothesis to be tested.

$H_0: P \leq 0.7$ vs $H_1: P > 0.7$

Under H_0 the test statistic is: $z = \dfrac{\hat{P} - P}{\sqrt{\dfrac{Pq}{n}}}$

$$\hat{P} = \frac{440}{600} = 0.73.$$

$$z = \frac{0.73 - 0.7}{\sqrt{\frac{0.7 \times 0.3}{600}}} = 1.60 \quad \text{and} \quad P(z > 1.60) = 0.0548.$$

At ~~5%~~ $\alpha = 0.05$, we fail to reject H_0. Hence the education/advertisement campaign was not effective.

15 $P = 0.39$ $q = 0.61$

$n = 55.$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

~~SD~~ Standard deviation $= 0.06577$

$z = \pm 0.196$

$\hat{p} = 0.5189, 0.2611$

Therefore, $x = 29, 15$

The sample recorded needs to show alcohol as a factor to suggest a significant difference exist between the national and local rate of alcohol related car accidents x is ~~less~~

$(15 \leq x \leq 29)$